instead of (3). Instead of being governed by the Mathieu equation, the TM wave is governed by the general Hill's equation [10]. However, for the case of $M\lambda/d \ll 1$, we can neglect the third term because it varies as $M/\lambda d$ while the first two terms vary as $1/\lambda^2$. We then have an equation identical in form to (3) and the problem can be solved by a parallel treatment.

It is worth pointing out that the modal theory provides the most rigorous approach to the problem of spatially periodic media. Retaining only two modes in the equations, we have derived the popular coupled wave equations from the postulate of small index modulation. The modal theory is shown to reduce to various previously arrived theories under simplified assumptions. Calculations are presented for cases when other theories fail to apply. We stress that the dispersion analysis techniques as discussed in this paper are very useful tools in studying wave behavior pertaining to periodic media. The paper is directed toward applications to optical components in integrated optics systems. Applications of the theory to other fields are topics worthwhile exploring.

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Generalized Analysis of Parallel Two-Post Mounting Structures in Waveguide

OSMAN L. EL-SAYED

***Abstract*—An analytical expression is obtained for the reactance of parallel two-post mounting structures having unsymmetrical strip and gap positions and different strip and gap widths. An equivalent circuit is derived and analytical expressions for its components established giving a physical insight into the problem of coupling between the two gaps. The analysis is based on deriving a variational expression for the structure reactance from the boundary conditions at the structure position. The theoretical results are experimentally verified for a wide range of coupling conditions.**

Exploitation of this model for the design of wide-band varactor-tuned negative resistance oscillators and multidiode parallel mounts in general is discussed.

Manuscript received December 23, 1975; revised May 25, 1976.

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List of Symbols

$J(r)$	Strip current density.
$I(y)$	Strip current.
I_g	Gap current.
v_g	Voltage drop across gap.
E_{in}	y-directed incident electric field intensity.
$E_r(r)$	y-directed scattered electric field intensity.
$E_g(r)$	Gap field intensity.
R	Reflection coefficient at mount position.
n	Free-space wave impedance equals $\sqrt{u_0/\varepsilon_0}$.
δ_n	Kronecker delta.
m,n	Mode number.
k_x	Equals $m\Pi/a$.
k_y	Equals $n\Pi/b$.
k_{p_m}	Post coupling coefficient equals $\sin k_x S(\sin(k_x W/2)/(k_x W/2))$.
k_{g_n}	Gap coupling coefficient equals $\cos k_y h(\sin(k_y g/2)/(k_y g/2))$.
λ	Free-space wavelength.
λ_g	Guide wavelength (dominant mode).
k	Free-space wave number equals $2\Pi/\lambda$.
Γ_{mn}	Guide wave number equals $\sqrt{k_x^2 + k_y^2 - k^2}$.
k'	Dominant wave number equals $(2\Pi/\lambda_g) = (\Gamma_{10}/j)$.
Z_0	Guide characteristic impedance equals $(2b/a)n(\lambda_g/\lambda)$.
$u(x)$	Distribution function given by

$$u(x) = \begin{cases} 1, & \text{for } S - \dfrac{W}{2} < x < S + \dfrac{W}{2} \\ 0, & \text{elsewhere.} \end{cases}$$

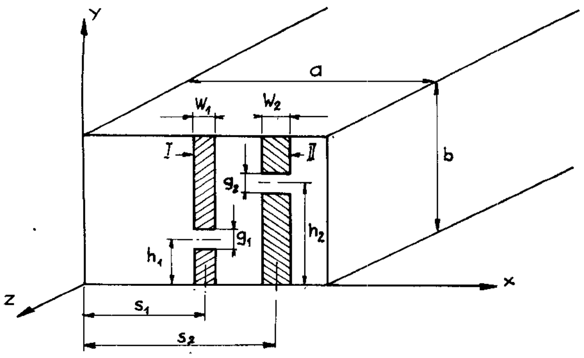


Fig. 1. Cross section of the rectangular guide showing the structure under study and its parameters.

$\theta(y)$ Distribution function given by

$$\theta(y) = \begin{cases} 1, & \text{for } h - \frac{g}{2} < y < h + \frac{g}{2} \\ 0, & \text{elsewhere.} \end{cases}$$

I,II,N Superscripts indicating that the quantity is respectively relevant to strip I, strip II, or both strips.

I. Introduction

PARALLEL two-post mounting structures are frequently used to couple two active or passive devices to a rectangular waveguide or cavity. In this type of mount both devices are mounted in separate gaps along two separate posts positioned in the same cross section of the guide.

A typical circuit in which this mounting structure is used is the broad-band varactor-tuned Gunn oscillator. It may also be used for multiple Gunn diode oscillators, p-i-n diode samplers, and commutators and varactor-tuned filters.

For the investigation of the mount configuration yielding the optimum coupling between the different elements, a lumped equivalent circuit for the mount is needed. The development of such a circuit will be our main concern here.

In a previous publication [1] we have studied the special case of the symmetrical mount where the posts are identical and symmetrically disposed with respect to the center of the guide, the gaps being at the same height. More recently, Chang and Khan [2] analyzed the coupling between two narrow transverse strips unsymmetrically located in the same cross section of a rectangular waveguide by extending to the two-post case the constant current density used by Collin [3]. In this paper, we treat the most general configuration of parallel double-post mounting structures by extending Lewin's analysis of a single post [4] to the double-post case.

II. Theoretical Analysis

The structure under study is shown in Fig. 1. The strips are assumed to be infinitesimally thin and perfectly conducting. Two impedances Z_d^{I} and Z_d^{II} are placed in the gaps of strips I and II, respectively.

In general, mounting structures may be tackled either as an antenna system radiating in the waveguide [1], [5] or as an obstacle in the waveguide [4]. In the following analysis the mount will be dealt with as an obstacle. A dominant mode (TE_{10}) wave is assumed to be incident on the mount, thus inducing, in general, two different currents in the posts. These currents, which are assumed to be uniformly distributed across the strips, will develop a voltage drop across each gap. Resulting gap fields will be assumed uniform all over the gaps.

A. Determination of Gap and Scattered Fields in Terms of Strip Currents

Since the strips are not uniform along the y direction, both strip currents will have a nonuniform distribution along the y direction. These current distributions are, *a priori*, unknown but will be expanded in a general orthogonal set of harmonic functions of period $2b$

$$I^N(y) = \sum_{n=0}^{\infty} (2 - \delta_n) I_n^N \cos k_y y, \qquad N = \mathrm{I,II}. \tag{1}$$

The current densities are then given by

$$J^{\mathrm{I}}(r) = \sum_{n=0}^{\infty} \frac{(2 - \delta_n)}{W_1} I_n^{\mathrm{I}} \cos k_y y \, u^{\mathrm{I}}(x)\delta(z) \tag{2a}$$

$$J^{\mathrm{II}}(r) = \sum_{n=0}^{\infty} \frac{2 - \delta_n}{W_2} I_n^{\mathrm{II}} \cos k_y y \, u^{\mathrm{II}}(x)\delta(z). \tag{2b}$$

Considering that the currents I_g^{I} and I_g^{II} flowing through the elements $(Z_d^{\mathrm{I}}, Z_d^{\mathrm{II}})$ present in the gaps are respectively equal to the average current in each gap, we can write

$$I_g^N = \sum_{n=0}^{\infty} (2 - \delta_n) I_n^N k_{g_n}^N. \tag{3}$$

The voltage drops and electric fields in the gaps are then given by

$$v_g^N = -I_g^N Z_d^N \tag{4}$$

$$E_g^{\mathrm{I}}(r) = \frac{I_g^{\mathrm{I}} Z_d^{\mathrm{I}}}{g_1} u^{\mathrm{I}}(x)\theta^{\mathrm{I}}(y)\delta(z) \tag{5a}$$

$$E_g^{\mathrm{II}}(r) = \frac{I_g^{\mathrm{II}} Z_d^{\mathrm{II}}}{g_2} u^{\mathrm{II}}(x)\theta^{\mathrm{II}}(y)\delta(z). \tag{5b}$$

The determination of the scattered y-directed electrical field $E(r)$ follows by using the expression

$$E_r(r) = -j\omega\mu_0 \int_{V'} G(r \mid r')[J^{\mathrm{I}}(r') + J^{\mathrm{II}}(r')] \, dV'$$

where $G(r \mid r')$ is the $\hat{y}$-$\hat{y}$ component of the dyadic Green's function for a rectangular guide [1]:

$$G(r \mid r') = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{(2 - \delta_n)(k^2 - k_y^2)}{abk^2\Gamma_{mn}} \cdot \sin k_x x \sin k_x x' \cos k_y y \cos k_y y' e^{-\Gamma_{mn}|z - z'|}.$$

This gives

$$E_r(r) = -j\frac{\eta}{ak}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{(2-\delta_n)(k^2-k_y^2)}{abk^2\Gamma_{mn}} \cdot [I_n^{\mathrm{I}}k_{pm}^{\mathrm{I}} + I_n^{\mathrm{II}}k_{pm}^{\mathrm{II}}]\sin k_x x \cos k_y y e^{-\Gamma_{mn}|z|}. \quad (6)$$

Let the y-directed electrical field of the incident TE_{10} wave (E_{in}) be given by

$$E_{\mathrm{in}} = \sin\left(\frac{\Pi x}{a}\right)e^{\Gamma_{10}z}. \quad (7)$$

The reflection coefficient R is then obtained by taking the ratio of the dominant mode back-scattered field to the incident field at the mount position

$$R = \left.\frac{E_r(n=0, m=1)}{E_{\mathrm{in}}}\right|_{z=0} = \frac{Z_0}{2b}[I_0^{\mathrm{I}}k_{p_1}^{\mathrm{I}} + I_0^{\mathrm{II}}k_{p_1}^{\mathrm{II}}]. \quad (8)$$

B. Boundary Conditions

In this section the boundary conditions are exploited, together with the relations established in Section II-A to derive a variational expression for the mount impedance as viewed from the waveguide, in terms of the currents spatial harmonics (which are still unknown).

The boundary conditions, at the mount, are such that the sum of the y-directed incident and scattered electrical fields vanishes all over the two strips except at the gaps where it is respectively equal to the gap fields.

$$[E_{\mathrm{in}} + E_r(r)]u^N(x)\delta(z) = E_g^N(r), \qquad N = \mathrm{I,II}.$$

Integrating the preceding equality with respect to x, over each post, we obtain, respectively,

$$k_{p_1}^{\mathrm{I}} - j\frac{\eta}{ak}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{(2-\delta_n)(k^2-k_y^2)}{\Gamma_{mn}} \cdot k_{pm}^{\mathrm{I}}[I_n^{\mathrm{I}}k_{pm}^{\mathrm{I}} + I_n^{\mathrm{II}}k_{pm}^{\mathrm{II}}]\cos k_y y = \frac{1}{g_1}I_g^{\mathrm{I}}Z_d^{\mathrm{I}}\theta^{\mathrm{I}}(y) \quad (9)$$

$$k_{p_1}^{\mathrm{II}} - j\frac{\eta}{ak}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{(2-\delta_n)(k^2-k_y^2)}{\Gamma_{mn}} \cdot k_{pm}^{\mathrm{II}}[I_n^{\mathrm{I}}k_{pm}^{\mathrm{I}} + I_n^{\mathrm{II}}k_{pm}^{\mathrm{II}}]\cos k_y y = \frac{1}{g_2}I_g^{\mathrm{II}}Z_d^{\mathrm{II}}\theta^{\mathrm{II}}(y). \quad (10)$$

It is to be noted that the integrations over x of the boundary condition equations were carried in order to bring out the dominant mode term in the form of the bracketed term in the right-hand side of (8).

In order to obtain a variational expression for the mount impedance, we multiply (9) and (10), respectively, by $I^{\mathrm{I}}(y)$ and $I^{\mathrm{II}}(y)$, integrate the resulting equations with respect to y from 0 to b, and add the two obtained equations. This gives

$$b[I_0^{\mathrm{I}}k_{p_1}^{\mathrm{I}} + I_0^{\mathrm{II}}k_{p_1}^{\mathrm{II}}] - \frac{Z_0}{2}[I_0^{\mathrm{I}}k_{p_1}^{\mathrm{I}} + I_0^{\mathrm{II}}k_{p_1}^{\mathrm{II}}]^2 - j\frac{\eta b}{ak}\sum_{n=0}^{\infty}{}'\sum_{m=1}^{\infty}\frac{(2-\delta\,)(k^2-k_y^2)}{\Gamma_{mn}} \cdot [I_n^{\mathrm{I}}k_{pm}^{\mathrm{I}} + I_n^{\mathrm{II}}k_{pm}^{\mathrm{II}}]^2 = I_g^{\mathrm{I}2}Z_d^{\mathrm{I}} + I_g^{\mathrm{II}2}Z_d^{\mathrm{II}} \quad (11)$$

where the prime between the summation symbols indicates that the term $n = 0, m = 1$ is excluded from the summation.

Dividing (11) by $Z_0[I_0^{\mathrm{I}}k_{p_1}^{\mathrm{I}} + I_0^{\mathrm{II}}k_{p_1}^{\mathrm{II}}]^2$ and substituting from (8), we obtain

$$-\frac{1}{2}\left(\frac{1+R}{R}\right) = j\frac{k'}{2}\sum_{n=0}^{\infty}{}'\sum_{m=1}^{\infty}\frac{(2-\delta_n)(1-k_y^2/k^2)}{\Gamma_{mn}} \cdot \left(\frac{I_n^{\mathrm{I}}k_{pm}^{\mathrm{I}} + I_n^{\mathrm{II}}k_{pm}^{\mathrm{II}}}{I_0^{\mathrm{I}}k_{p_1}^{\mathrm{I}} + I_0^{\mathrm{II}}k_{p_1}^{\mathrm{II}}}\right)^2 + \frac{I_g^{\mathrm{I}2}(Z_d^{\mathrm{I}}/Z_0) + I_g^{\mathrm{II}2}(Z_d^{\mathrm{II}}/Z_0)}{(I_0^{\mathrm{I}}k_{p_1} + I_0^{\mathrm{II}}k_{p_1}^{\mathrm{II}})^2}. \quad (12)$$

It can be readily shown that the left-hand side of (12) is equal to the normalized obstacle impedance $z_{\mathrm{ob}} = (Z_{\mathrm{ob}}/Z_0)$. Moreover, if we refer all the quantities to the center of the guide and normalize the impedances, (12) can be rewritten as

$$z_{\mathrm{ob}} = j\frac{k'}{2}\sum_{n=0}^{\infty}{}'\sum_{m=1}^{\infty}\frac{(2-\delta_n)(1-k_y^2/k^2)}{\Gamma_{mn}} \cdot \left[\frac{I_n^{\mathrm{I}'}(k_{pm}^{\mathrm{I}}/k_{p_1}) + I_n^{\mathrm{II}'}(k_{pm}^{\mathrm{II}}/k_{p_1}^{\mathrm{II}})}{I_0^{\mathrm{I}'} - I_0^{\mathrm{II}'}}\right]^2 + \frac{(I_g^{\mathrm{I}'})^2 z_d^{\mathrm{I}'} + (I_g^{\mathrm{II}'})^2 z_d^{\mathrm{II}'}}{(I_0^{\mathrm{I}'} + I_0^{\mathrm{II}'})^2} \quad (13)$$

where

$$I_n^{N'} = I_n^N k_{p_1}^N, \qquad n = 0,1,2,\cdots,\infty, \quad N = \mathrm{I,II}. \quad (14)$$

$$I_g^{N'} = I_g^N k_{p_1}^N \quad (15)$$

$$z_d^{N'} = \frac{Z_d^N}{Z_0(k_{p_1}^N)^2}. \quad (16)$$

C. Evaluation of the Currents Spatial Harmonics and Obstacle Impedance

In order to express the mount impedance only in terms of frequency and the mount geometrical parameters, the different currents spatial harmonics should be evaluated and substituted in (13). For this we proceed as follows.

1) By exploiting the orthogonality of the different terms of (9) and (10), we obtain expressions for the different currents spatial harmonics ($I_n^{\mathrm{I,II}}$) in terms of the voltage drops across the gaps v_g^{I} and v_g^{II}.

2) Referring to (3) and (4), we can see that the gaps voltage drops are themselves expressed in terms of the currents spatial harmonics. Thus, substituting by the expres-

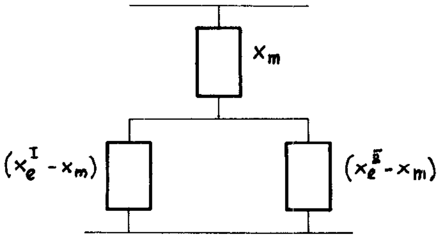


Fig. 2. Equivalent circuit for the structure (referred to the center of the guide) when the two gaps are short circuited.

sions obtained in step 1), in (4) we can express the gap voltage drops in terms of the mount parameters.

3) Substituting again by the expressions obtained in step 2) for $v_g^{I,II}$ into those found in 1), we obtain the required expressions for the currents spatial harmonics and hence the final expression for z_{ob}.

The details of the aforementioned mathematical manipulations can be found in the Appendix and can be seen to lead to the following expression:

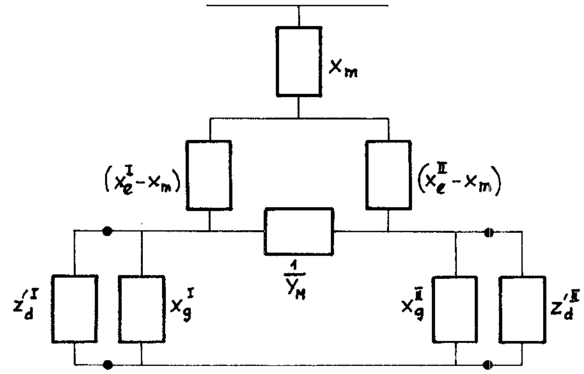


Fig. 3. Equivalent circuit for the structure under study (referred to the center of the guide.)

coupling network corresponding to all $n > 0$ with the coupling network corresponding to $n = 0$ (Fig. 3).

It can be readily seen that the obtained expression for z_{ob} [see (17)] fits the equivalent circuit just described. This

$$z_{ob} = x_m + \frac{(x_e^I - x_m)(x_e^{II} - x_m)}{(x_e^I - x_m) + (x_e^{II} - x_m) + 1/Y_M}$$

$$+ \frac{\left[\frac{1}{Y_g^I} + \frac{(x_e^I - x_m)\cdot(1/Y_M)}{(x_e^I - x_m) + (x_e^{II} - x_m) + (1/Y_M)}\right]\left[\frac{1}{Y_g^{II}} + \frac{(x_e^{II} - x_m)\cdot(1/Y_M)}{(x_e^I - x_m) + (x_e^{II} - x_m) + (1/Y_M)}\right]}{\left[\frac{1}{Y_g^I} + \frac{(x_e^I - x_m)\cdot(1/Y_M)}{(x_e^I - x_m) + (x_e^{II} - x_m) + (1/Y_M)}\right] + \left[\frac{1}{Y_g^{II}} + \frac{(x_e^{II} - x_m)\cdot(1/Y_M)}{(x_e^I - x_m) + (x_e^{II} - x_m) + (1/Y_M)}\right]}. \quad (17)$$

D. Determination of Equivalent Circuit Configuration

In order to determine the configuration of the equivalent circuit, we will first consider the special case where the two gaps are short circuited ($z_d^{I'} = z_d^{II'} = 0$). It can be readily seen that in this case

$$I_n^{I'} = I_n^{II'} = 0, \qquad \text{for all } n > 0$$

$$I_g^{I'} = I_0^{I'} = \frac{v_0}{D_0}(x_e^{II} - x_m)$$

$$I_g^{II'} = I_0^{II'} = \frac{v_0}{D_0}(x_e^{I} - x_m)$$

$$z_{ob} = x_m + \frac{(x_e^I - x_m)(x_e^{II} - x_m)}{x_e^I + x_e^{II} - 2x_m}.$$

The preceding equations are seen to be satisfied by the equivalent circuit of Fig. 2, which is the same as that found by Chang and Khan [2]. It represents the coupling between the posts due to all the modes $n = 0$, $m = 1,2,\cdots,\infty$, x_m being the coupling impedance and x_e^I, x_e^{II} the impedance of each post, respectively, when the other is absent.

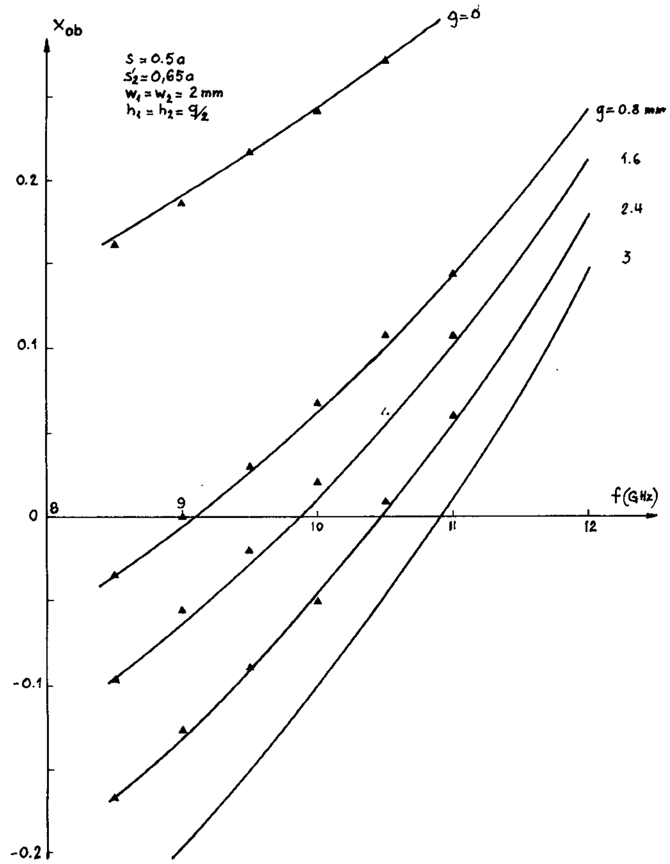
In the general case, beside the coupling due to the modes $n = 0$, further coupling occurs due to all the modes $n > 0$. This can be accounted for by an infinite number of Π coupling networks connected in parallel between the two gaps, each one corresponding to a value of n [1]. The complete equivalent circuit is obtained by combining the resultant Π enables us to give the analytical expressions for each of its elements.

A further evidence of the validity of this equivalent circuit is brought by the fact that the same configuration was found in [1] although the problem was dealt with as a system of coupled antennas. We can also verify that the expressions for the different components reduce to those obtained in [1] for the special case of symmetrical mount.

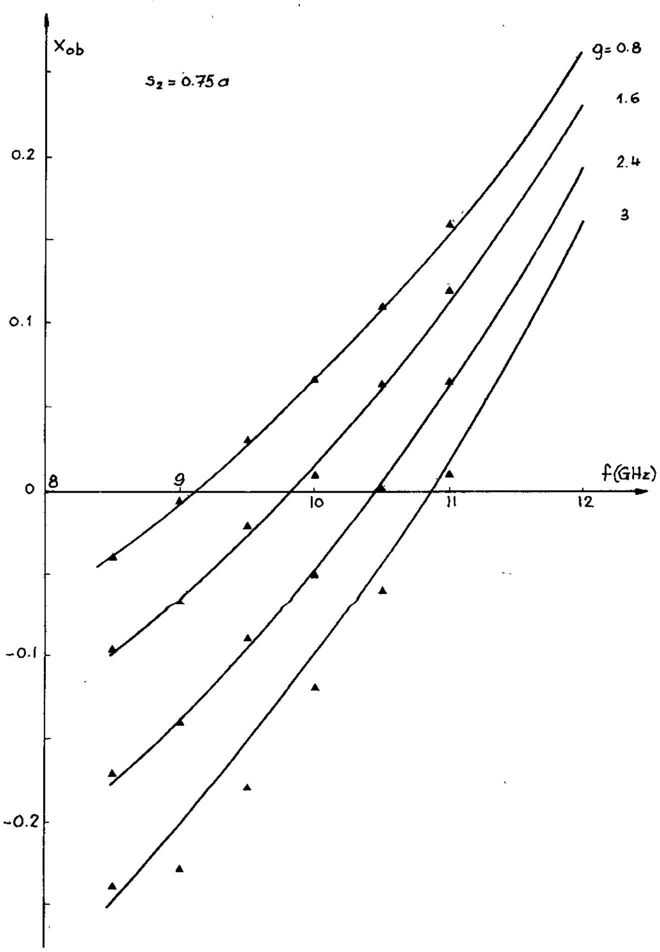
III. Experimental Verification

Measurements of the mount impedance, as viewed from the waveguide port, were carried out in conventional X-band rectangular guide, over a wide frequency range for different combinations of strip and gap positions. The results were compared to theoretical values.

In Fig. 4(a)–(c) strip I was kept in the same position ($S_1/a = 0.5$) while strips II were respectively positioned at $S_2/a = 0.65$, 0.75, and 0.85. In the three cases the gaps were positioned at the bottom of the guide and had the same widths $g_1 = g_2 = 0.8$, 1.6, 2.4, and 3 mm. In Fig. 5 each of the two gaps was alternatively short circuited while the other remained open ($S_1/a = 0.5$, $S_2/a = 0.85$). These configurations correspond to the cases $Z_d^I = 0$, $Z_d^{II} = \infty$ and vice versa. In Fig. 6, the gap of strip I was maintained at the bottom of the guide while the height h_2 of the other gap was varied between $0.12b$ and $0.88b$, the gap widths being equal. On the whole, experimental results show a good agreement with theoretical ones over a wide range of coup-



(a)



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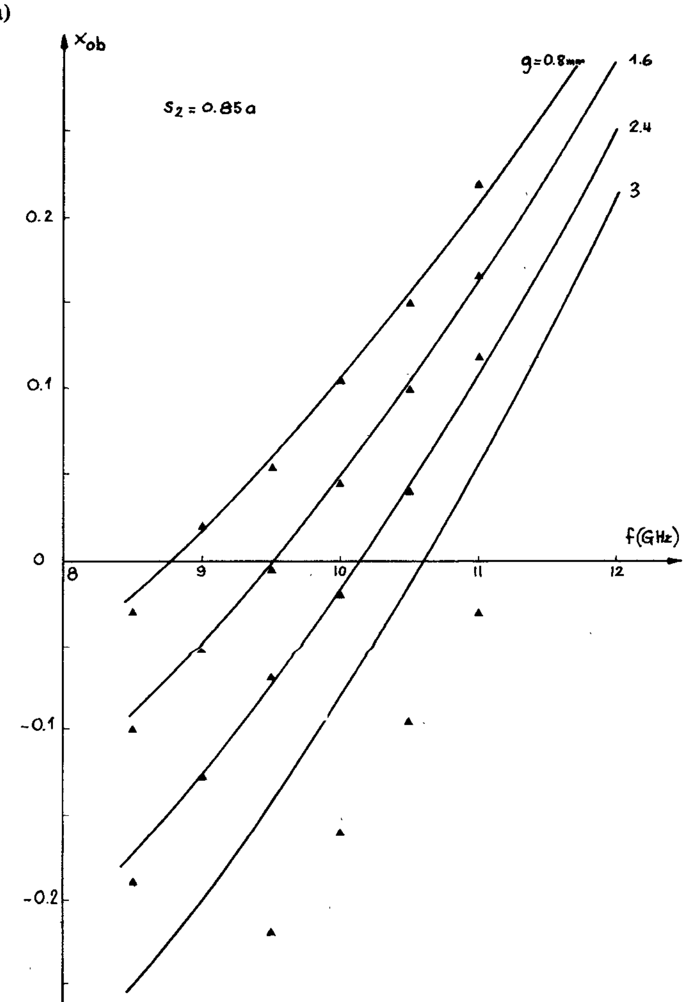

(c)

Fig. 4. Obstacle reactance as a function of frequency for different positions of strip II: $S_1/a = 0.5$; $W_1 = W_2 = 2$ mm; $g_1 = g_2 = g$; $h_1 = h_2 = g/2$. (a) $S_2/a = 0.65$. (b) $S_2/a = 0.75$. (c) $S_2/a = 0.85$. ○ Experimental point. — Theoretical curve.

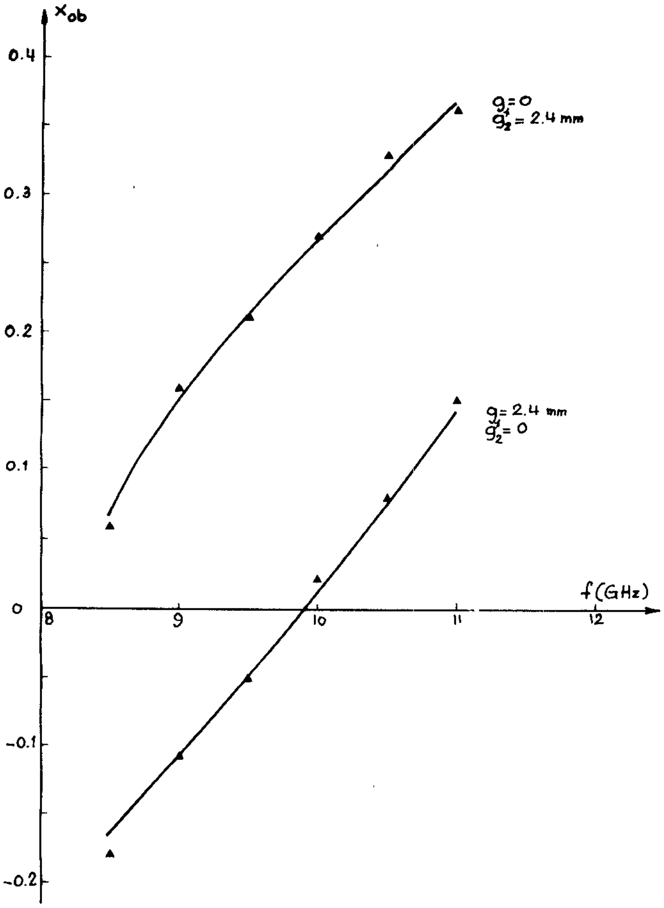

Fig. 5. Obstacle reactance as a function of frequency for different values of gap impedances ($Z_d'^{\mathrm{I}}$ and $Z_d'^{\mathrm{II}}$). ○ Experimental point. — Theoretical curve.

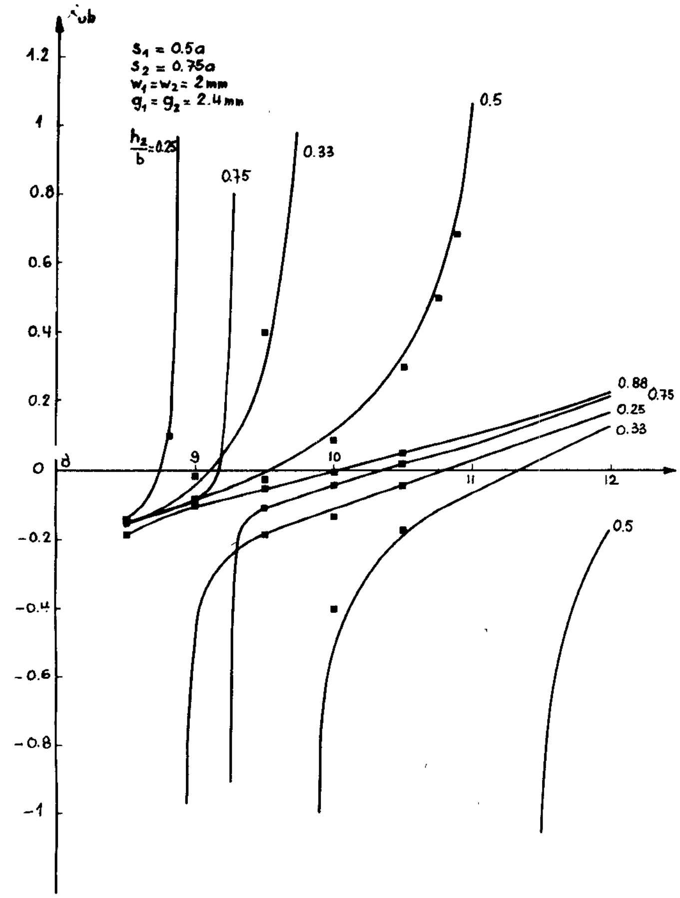

Fig. 6. Obstacle reactance as a function of a frequency for different values of gap height. $S_1/a = 0.5$; $S_2/a = 0.75$; $W_1 = W_2 = 2$ mm; $g_1 = g_2 = 2.4$ mm; $h_1/b = 0.12$; $h_2/b = 0.25, 0.33, 0.5, 0.88$.

ling conditions as long as the gap widths (g_1,g_2) do not exceed $b/4$.

It must be noted that the mount configurations tested were chosen so that the high-order components of the equivalent circuit (x_g^{I}, x_g^{II}, $1/Y_M$) are not masked from the waveguide port by high impedance values for x_e^{I}, x_e^{II}, and x_m. This is specially the case in Fig. 6 where the anti-resonance frequency is very sensitive to the values of the high-order components.

IV. Investigation of Optimum Mount Configurations

Although the analysis has been conducted for two flat strips, the results can be extended to the case of two cylindrical posts through the introduction of an equivalence relation between strip widths (W) and post diameters (d) of the form $W = cd$ where c is an experimentally determined constant [1], [5]. In addition, two identical series reactances should be introduced in series to account for the phase difference between reflected and transmitted waves due to the finite dimensions of the posts in the z direction [1].

The large variety of possible configurations implies that the experimental constants should be determined each time for a definite configuration.

However, the value of the established equivalent circuit lies in providing a good physical insight into the problem

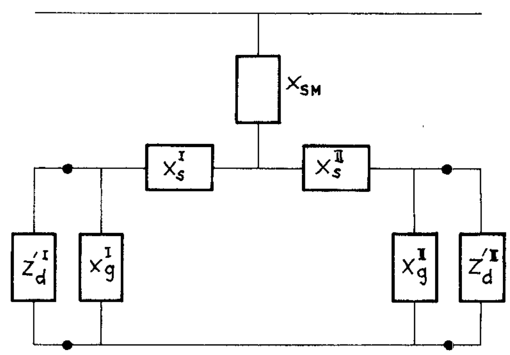

Fig. 7. Simplified form of obstacle equivalent circuit.

of coupling two diodes in a parallel mount, thus permitting the investigation of optimum mount configurations for multidiode circuits.

Let us investigate, for instance, the optimum configuration, for broad-band varactor-tuned Gunn oscillators. In order to do so, the equivalent circuit of Fig. 3 is reduced to that of Fig. 7 by a Δ/Y transformation, and the values of the components x_S^{I}, x_S^{II}, and x_{S_M} computed for different post and gap coupling conditions (Figs. 8–10).

In general, wide-band tuning is achieved when x_S^{I}, x_S^{II}, and x_{S_M} are small and do not show large variations within the frequency band of interest. Inspection of Figs. 8–10

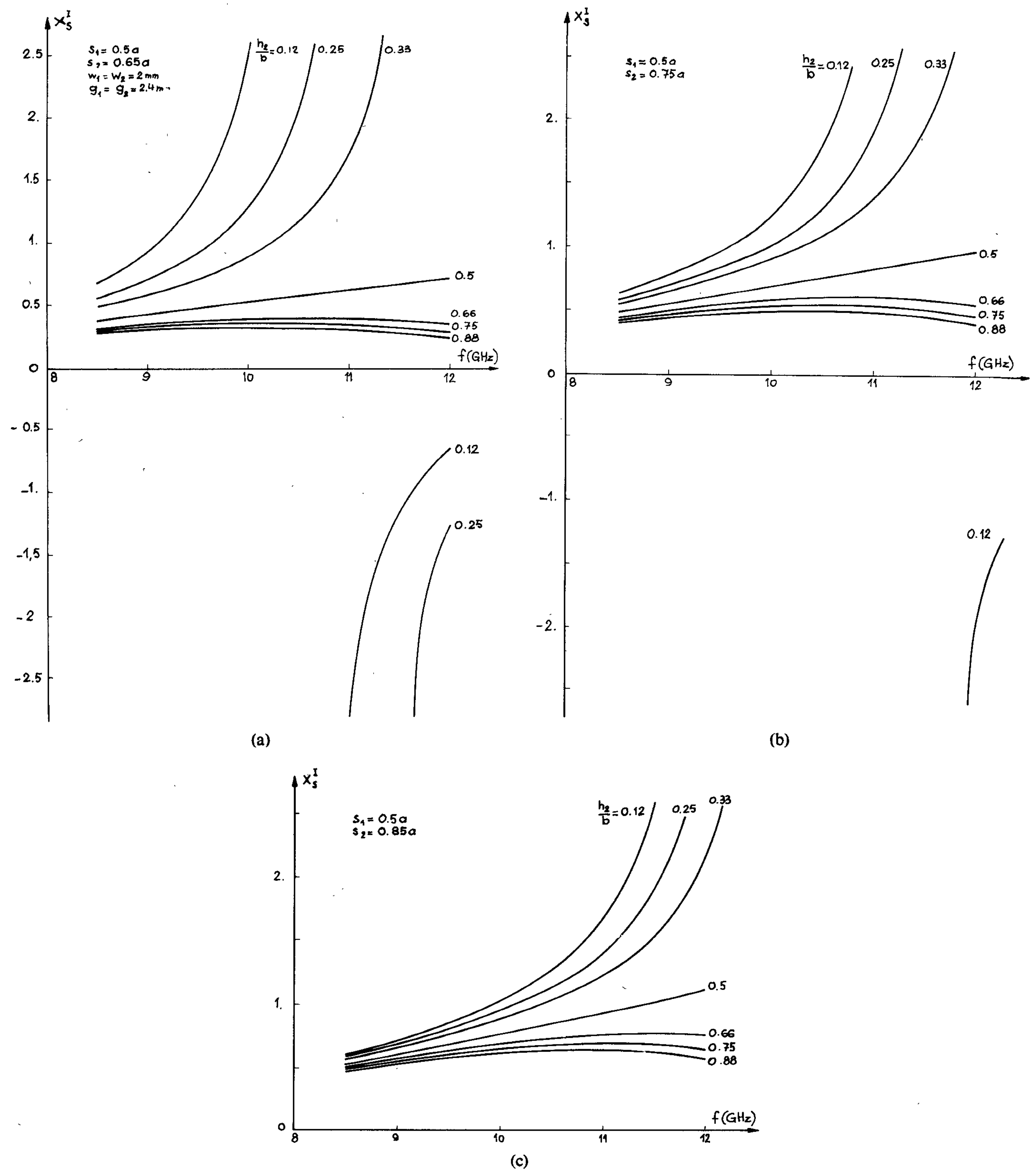

Fig. 8. Strip I series reactance ($x_S{}^I$) as a function of frequency for different values of gap II height (h_2). $S_1/a = 0.5$; $W_1 = W_2 = 2$ mm; $g_1 = g_2 = 2.4$ mm; $h_1/b = 0.12$; $h_2/b = 0.12 \rightarrow 0.88$. (a) $S_2/a = 0.65$. (b) $S_2/a = 0.75$. (c) $S_2/a = 0.85$.

shows that these conditions are fulfilled over the whole X band when the posts are closely coupled around the center of the guide with one gap at the bottom of one post and the other gap at the top of second post. This holds whether the posts are symmetrical or not with respect to the guide center. This confirms the experimental results obtained by Joshi [6]. This same configuration can also be used for coupling two identical Gunn diodes to the same cavity.

The parallel double-post mount can also be exploited for the design of mechanically and/or electronically tuned

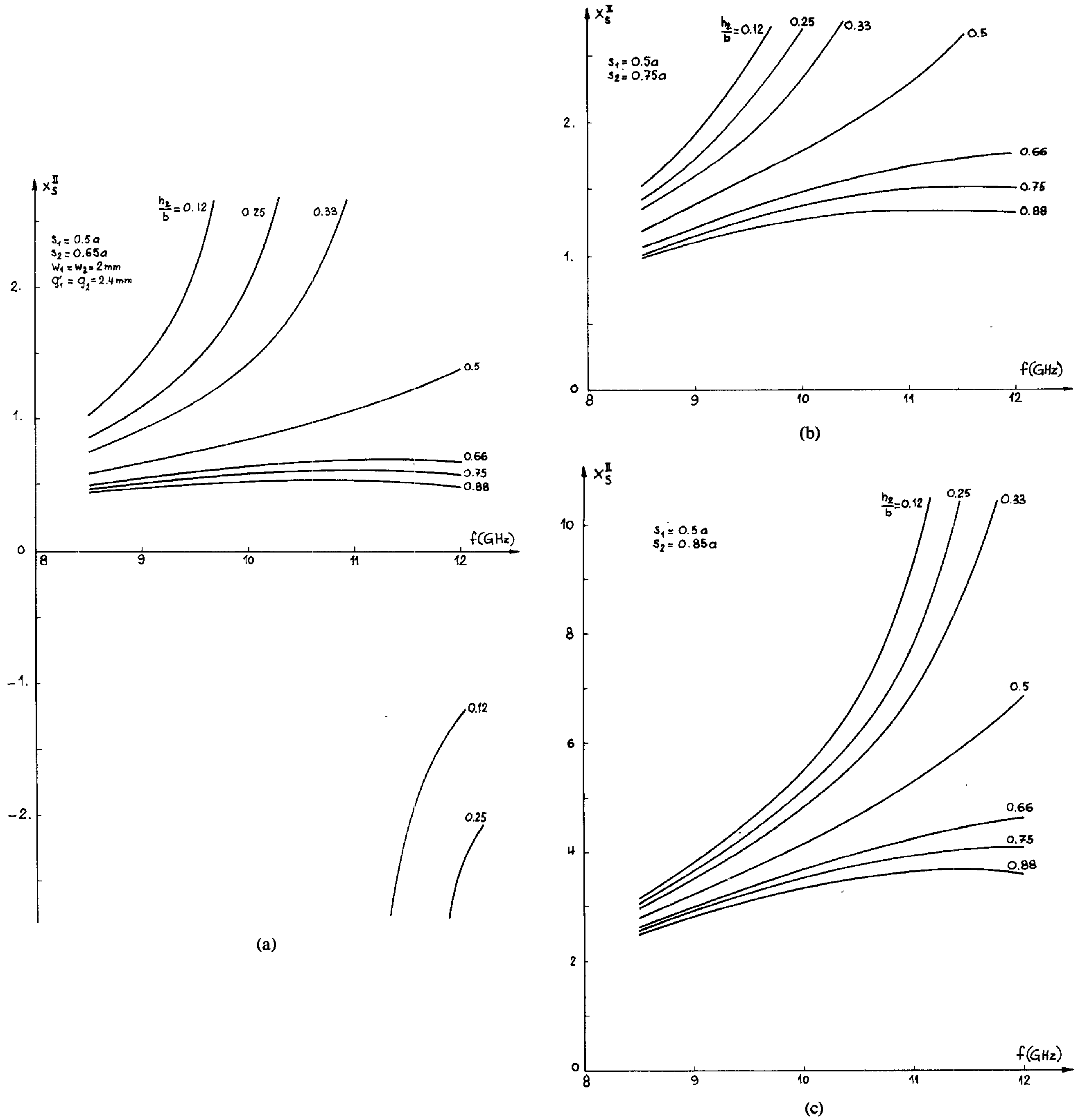

Fig. 9. Strip II series reactance (x_S^{II}) as a function of frequency for different values of gap II height (h_2). $S_1/a = 0.5$; $W_1 = W_2 = 2$ mm; $g_1 = g_2 = 2.4$ mm; $h_1/b = 0.12$; $h_2/b = 0.12 \rightarrow 0.88$. (a) $S_2/a = 0.65$. (b) $S_2/a = 0.75$. (c) $S_2/a = 0.85$.

bandpass filters. To obtain a bandpass, the two branches $(x_S^I + x_g^I)$ and $(x_S^{II} + x_g^{II})$ of the equivalent circuit (Fig. 7) should resonate in parallel at the required frequency, the impedance of the mount being small outside the passband. This performance can be obtained for any frequency in the band with a configuration similar to that tested in Fig. 6 where we can see that by changing gap II height (or its width) we can obtain coarse (or fine) mechanical tuning. If in gap II a varactor is introduced, electronic tuning can be achieved.

The preceding examples, among many others, demonstrate the usefulness of the obtained model in the design and the optimization of multidiode circuits in rectangular waveguide.

V. Conclusion

A variational expression for the "obstacle" impedance of the most general configuration of parallel double-post mounting structures has been established. Starting from this

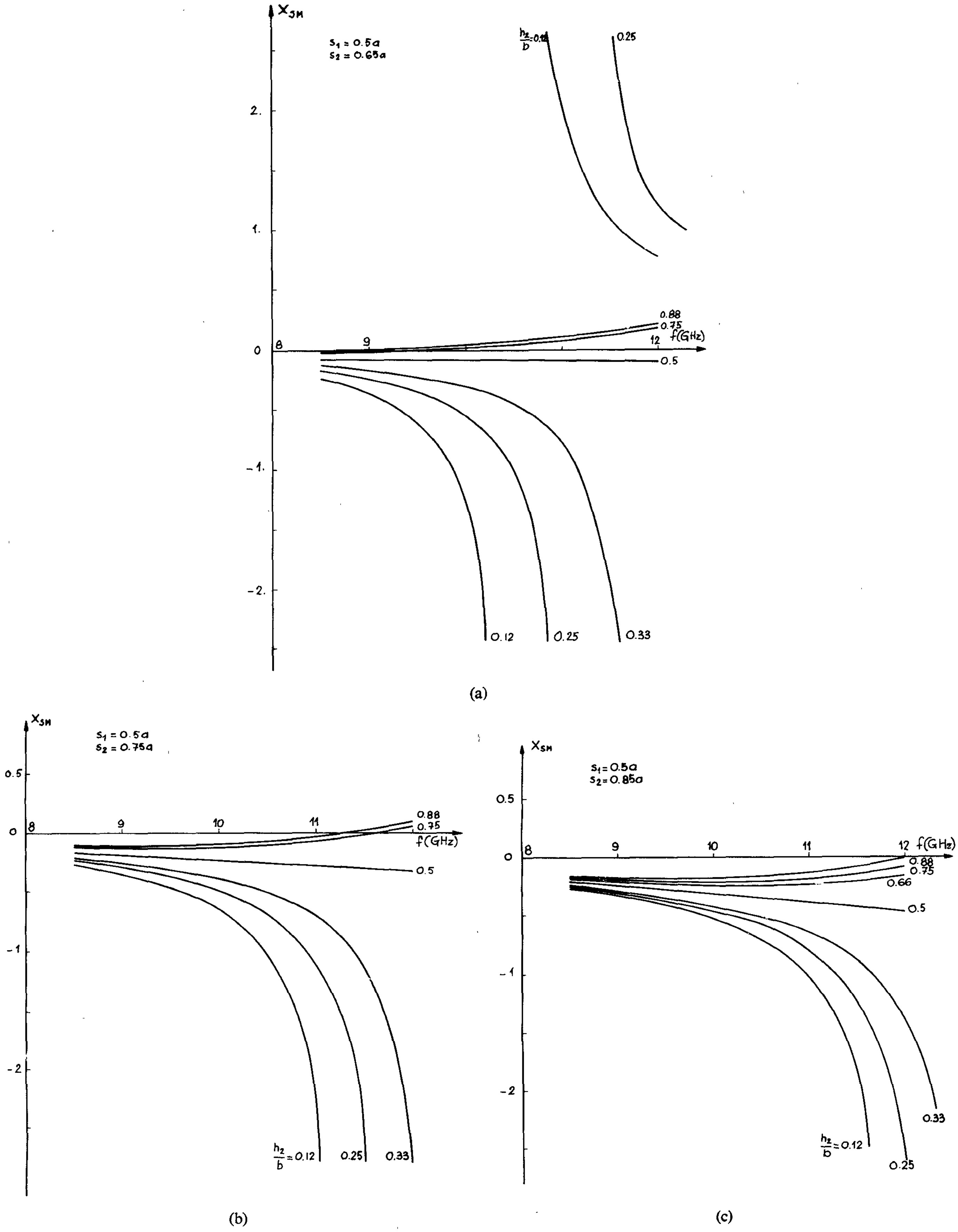

Fig. 10. Guide coupling reactance (x_{SM}) as a function of frequency for different values of gap II height (h_2). $S_1/a = 0.5$; $W_1 = W_2 = 2$ mm; $g_1 = g_2 = 2.4$ mm; $h_1 b = 0.12$; $h_2/b = 0.12 \rightarrow 0.88$. (a) $S_2/a = 0.65$. (b) $S_2/a = 0.75$. (c) $S_2/a = 0.85$.

expression, a lumped equivalent circuit has been developed and analytical expressions for its components derived.

Theoretical results were experimentally verified over a wide range of frequencies and coupling conditions. Optimum mount configurations for wide-band varactor-tuned Gunn oscillators, double-diode Gunn oscillators, and mechanically (or electronically) tuned bandpass filters were discussed, demonstrating thereby the usefulness of the model as a tool for the design and optimization of multidiode circuits in rectangular waveguide.

Appendix

Let us define the following quantities:

$$x_e^N = j\frac{k'}{2}\sum_{m=2}^{\infty}\frac{(k_{p_m}{}^N/k_{p_1}{}^N)^2}{\Gamma_{m0}} \quad \text{(A1)}$$

$$x_m = j\frac{k'}{2}\sum_{m=2}^{\infty}\frac{(k_{p_m}{}^{\rm I}k_{p_m}{}^{\rm II}/k_{p_1}{}^{\rm I}k_{p_1}{}^{\rm II})}{\Gamma_{m0}} \quad \text{(A2)}$$

$$x_{g_n}{}^N = j\frac{k'}{2}\sum_{m=1}^{\infty}\frac{1-k_y{}^2/k^2}{\Gamma_{mn}}\left(\frac{k_{p_m}{}^N/k_{p_1}{}^N}{k_{g_n}{}^N}\right)^2 \quad \text{(A3)}$$

$$x_{M_n} = j\frac{k'}{2}\sum_{m=1}^{\infty}\frac{1-k_y{}^2/k^2}{\Gamma_{mn}}\frac{(k_{p_m}{}^{\rm I}\cdot k_{p_m}{}^{\rm II})/(k_{p_1}{}^{\rm I}\cdot k_{p_1}{}^{\rm II})}{k_{g_n}{}^{\rm I}k_{g_n}{}^{\rm II}} \quad \text{(A4)}$$

$$z_e^N = \tfrac{1}{2} + x_e^N \quad \text{(A5)}$$

$$z_m = \tfrac{1}{2} + x_m \quad \text{(A6)}$$

$$v_0 = \frac{b}{z_0} \quad \text{(A7)}$$

$$v_g^{N'} = -I_g^{N'}z_d^{N'} = -\left[\sum_{n=0}^{\infty}(2-\delta_n)I_n^{N'}k_{g_n}{}^N\right]z_d^{N'} \quad \text{(A8)}$$

where $N = \text{I,II}$.

1) Substituting by (14)–(16) and (A1)–(A8) into (9) and (10), we obtain, respectively,

$$I_0^{\rm I'}z_e^{\rm I} + I_0^{\rm II'}z_m + 2\sum_{n=1}^{\infty}x_{g_n}{}^{\rm I}I_n^{\rm I'}k_{g_n}{}^{\rm I2}\cos k_y y + 2\sum_{n=1}^{\infty}x_{M_n}I_n^{\rm II'}k_{g_n}{}^{\rm I}k_{g_n}{}^{\rm II}\cos k_y y = v_0 + \frac{b}{g_1}v_g^{\rm I'}\theta^{\rm I}(y) \quad \text{(A9)}$$

$$I_0^{\rm II'}z_e^{\rm II} + I_0^{\rm I'}z_m + 2\sum_{n=1}^{\infty}x_{g_n}{}^{\rm II}I_n^{\rm II'}k_{g_n}^{\rm II2}\cos k_y y + 2\sum_{n=1}^{\infty}x_{M_n}I_n^{\rm I'}k_{g_n}{}^{\rm I}k_{g_n}{}^{\rm II}\cos k_y y = v_0 + \frac{b}{g_2}v_g^{\rm II'}\theta^{\rm II}(y). \quad \text{(A10)}$$

Integrating (A9) and (A10) over y from 0 to b and solving the resultant equations for $I_0^{\rm I'}$ and $I_0^{\rm II'}$, we obtain

$$I_0^{\rm I'} = \frac{v_0}{D_0}(z_e^{\rm II} - z_m) + \frac{v_g^{\rm I'}z_e^{\rm II} - v_g^{\rm II'}z_e^{\rm I}}{D_0} \quad \text{(A11)}$$

$$I_0^{\rm II'} = \frac{v_0}{D_0}(z_e^{\rm I} - z_m) + \frac{v_g^{\rm II'}z_e^{\rm I} - v_g^{\rm I'}z_e^{\rm II}}{D_0} \quad \text{(A12)}$$

where $D_0 = z_e^{\rm I}z_e^{\rm II} - z_m^2$.

Similarly, if (A9) and (A10) are multiplied by $\cos k_y y$, integrated over y from 0 to b and the resultant equations solved for $I_n^{\rm I'}$ and $I_n^{\rm II'}$, we obtain

$$I_n^{\rm I'}k_{g_n}{}^{\rm I} = \frac{v_g^{\rm I'}x_{g_n}{}^{\rm II} - v_g^{\rm II'}x_{M_n}}{D_n} \quad \text{(A13)}$$

$$I_n^{\rm II'}k_{g_n}{}^{\rm II} = \frac{v_g^{\rm II'}x_{g_n}{}^{\rm I} - v_g^{\rm I'}x_{M_n}}{D_n} \quad \text{(A14)}$$

where $D_n = x_{g_n}{}^{\rm I}x_{g_n}{}^{\rm II} - x_{M_n}{}^2$.

2) Substituting by (A11)–(A15) into (A8), we obtain two simultaneous equations for $v_g^{\rm I'}$ and $v_g^{\rm II'}$ which when solved together give

$$v_g^{\rm I'} = -\frac{v_0}{D_0}\frac{(z_e^{\rm II} - z_m)Y_2 + (z_e^{\rm I} - z_m)Y_3}{F} \quad \text{(A15)}$$

$$v_g^{\rm II'} = -\frac{v_0}{D_0}\frac{(z_e^{\rm I} - z_m)Y_1 + (z_e^{\rm II} - z_m)Y_3}{F} \quad \text{(A16)}$$

where

$$Y_1 = \frac{1}{z_d^{\rm I'}} + \frac{z_e^{\rm II}}{D_0} + \sum_{n=1}^{\infty}2\frac{x_{g_n}{}^{\rm II}}{D_n}$$

$$Y_2 = \frac{1}{z_d^{\rm II'}} + \frac{z_e^{\rm I}}{D_0} + \sum_{n=1}^{\infty}2\frac{x_{g_n}{}^{\rm I}}{D_n}$$

$$Y_3 = \frac{z_m}{D_0} + \sum_{n=1}^{\infty}2\frac{x_{M_n}}{D_n}$$

$$F = Y_1Y_2 - Y_3^2.$$

3) Substituting with the expressions obtained for $v_g^{\rm I'}$ and $v_g^{\rm II'}$ into (A8), (A11)–(A14), we obtain the required expressions for $I_g^{\rm I'}$, $I_g^{\rm II'}$, $I_0^{\rm I'}$, $I_0^{\rm II'}$, $I_n^{\rm I'}$, and $I_n^{\rm II'}$.

Finally, we substitute into (13) by the obtained expressions for the gap and spatial harmonic currents, and (17) is obtained in which

$$Y_g^{\rm I} = \frac{1}{z_d^{\rm I'}} + 2\sum_{n=1}^{\infty}\frac{x_{g_n}{}^{\rm II} - x_{M_n}}{D_n} \quad \text{(A17)}$$

$$Y_g^{\rm II} = \frac{1}{z_d^{\rm II'}} + 2\sum_{n=1}^{\infty}\frac{x_{g_n}{}^{\rm I} - x_{M_n}}{D_n} \quad \text{(A18)}$$

$$Y_M = 2\sum_{n=1}^{\infty}\frac{x_{M_n}}{D_n}. \quad \text{(A19)}$$

Acknowledgment

The author wishes to thank Prof. J. Hervé for his valuable suggestions and for his continuous encouragement. He also wishes to thank G. Mailhan for his technical assistance.

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